Bottleneck in the Development of Quantum Computers

QUC conference

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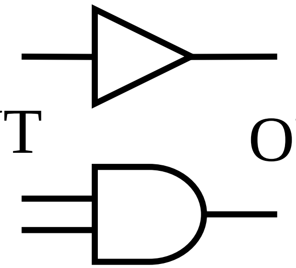
Outline

1. What is Quantum Computing?
2. How are Quantum Algorithms implemented?
3. NMR QC
4. Ex) Deutsch algorithm
5. Bottleneck of QC

1. What is Quantum computing?

Classical computing



Quantum computing

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

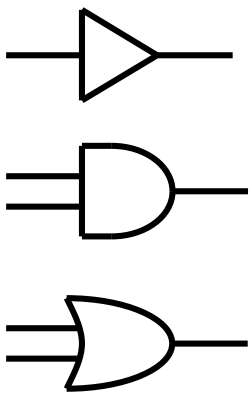
$$\boxed{\psi} = \boxed{e^{-iHt/\hbar}}\ \boxed{\psi_0}$$

OUTPUT **U** INPUT

	Classical computing	Quantum computing
Bit state	0V & 5V states	Quantum eigenstates –***suposition*** Ex)spin up & down Photon polarization
operation	semiconductor Gates	***Unitary operation***
Algorithm Excution	Spatial array of physical gates	Serial excution of unitary operations

Classical computing

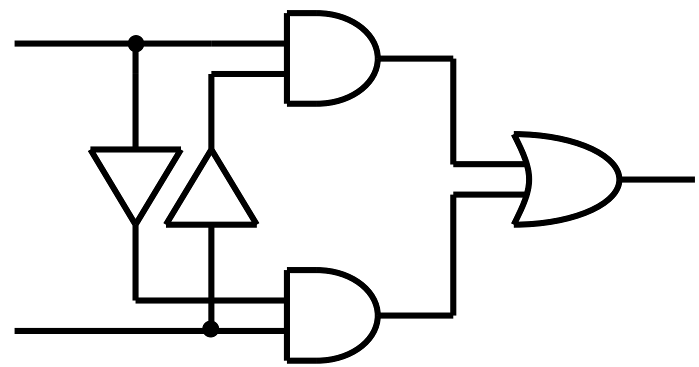
INPUT



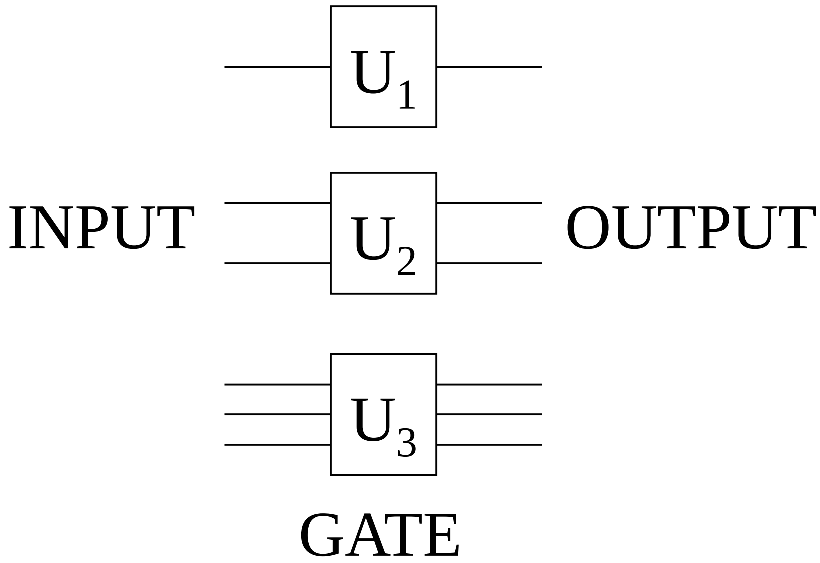
OUTPUT

GATE

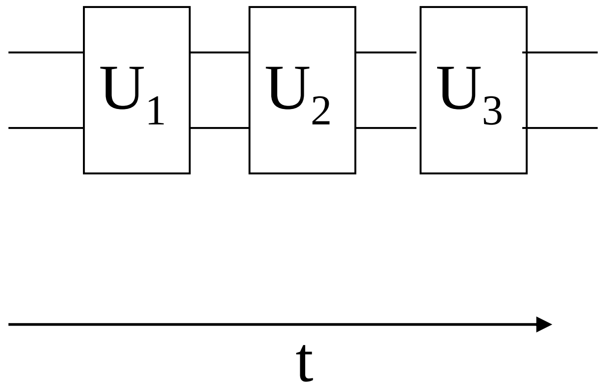
Ex) ADDER

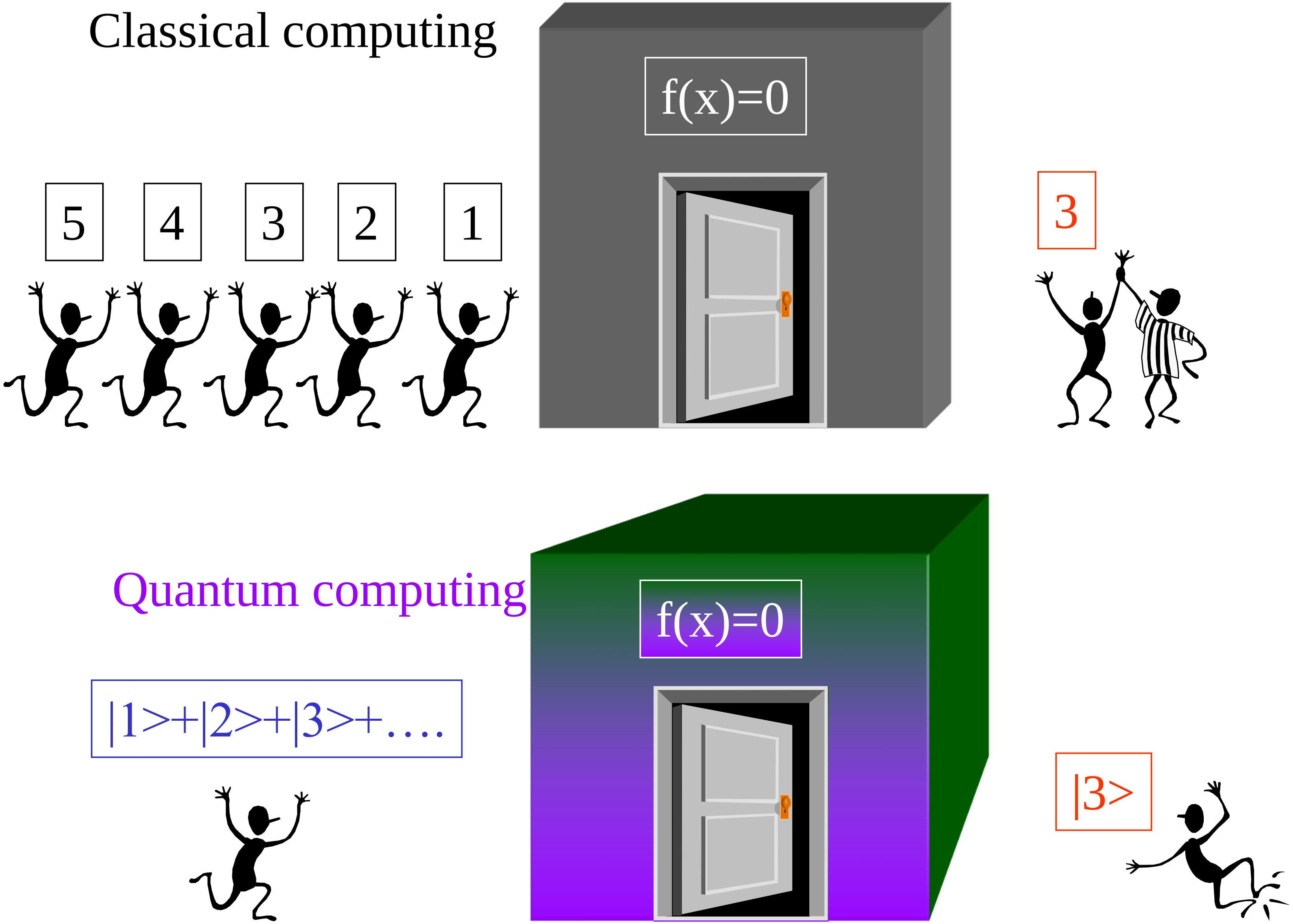


Quantum computing



Ex)




Classical computing
f(x)=0
5
4
3
2
1
3
Quantum computing
f(x)=0
|1>+|2>+|3>+….
|3>

2. How are Quantum Algorithms implemented?

- Classical computation

  : Algorithm-program-machine code -physical gates

- Quantum computation

  : Algorithm-unitary operation-gate operation- physical operations

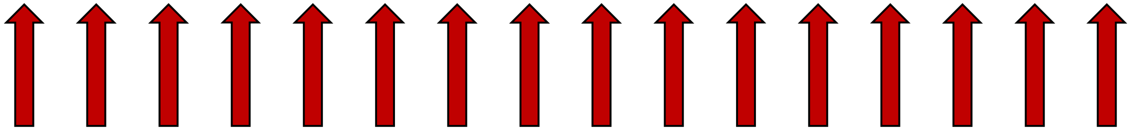
qubit - two states with good quantum

- charge : quantum dot
- spin : quantum dot, molecular magnet, ion trap, NMR, Si-based QC
- photon : optical QC, cavity QED
- cooper pair : superconductor
- fluxoid : superconductor

Spin qubit quantum computer

- Hamiltonian – Zeeman & interaction terms.

$$H = \sum_i \hbar\omega_i I_{i\alpha} + \sum_{i,j} J_{ij} I_{iz} I_{jz}$$



Execution of quantum algorithm

(1) Programming - unitary operator U + measurements

(2) Assemble - Decomposition of U : $U=U_1U_2U_3\ldots$

where U_i is a gate operation.

(3) Compile - Realization of gates by physical operations

Execution of quantum algorithm

(1) Programming - unitary operators + measurements

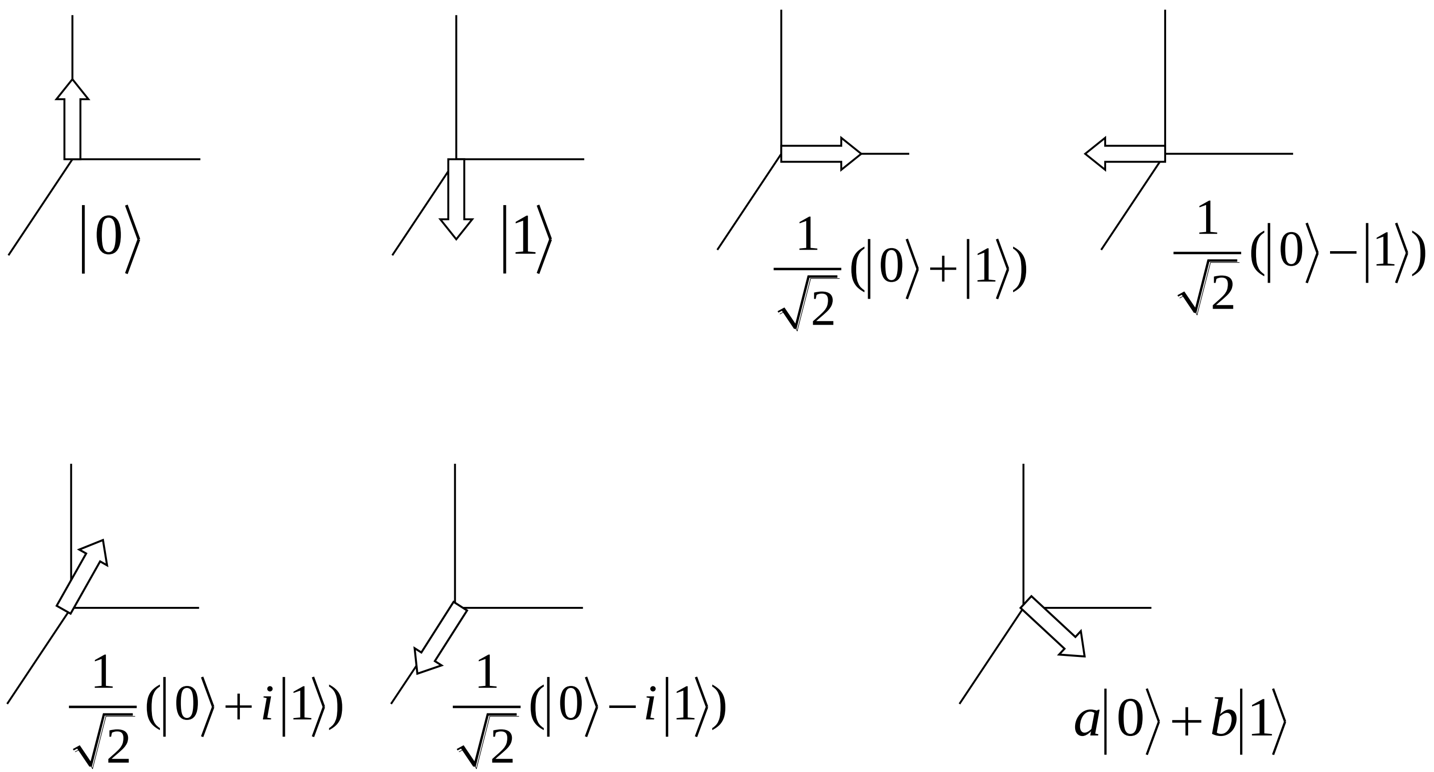
(2) Assemble - Decomposition of U : $U=U_1U_2U_3\ldots$

where U_i is a basic (gate) operation.

"Any unitary operator can be expressed as a combination of single qubit operators and controlled-NOT operators."

(3) Compile - Realization of gates by physical operations

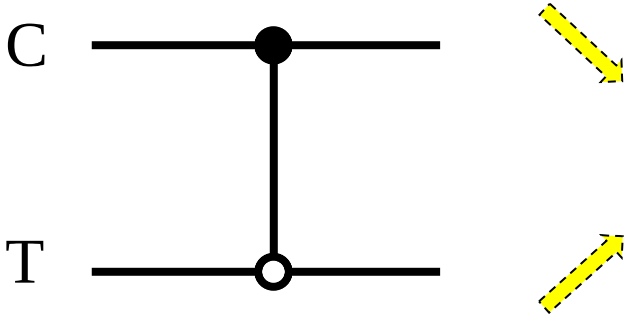
Single qubit operations of spin qubits
(Bloch Sphere representation)



Infinitely many operations

Controlled-NOT

input		output	
C	T	C	T
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0



$U(|0\rangle + |1\rangle)|0\rangle$: disentangled state

$= |0\rangle|0\rangle + |1\rangle|1\rangle$: entangled state

Execution of quantum algorithm

(1) Programming - unitary operators + measurements

(2) Assemble - Decomposition of U : $U=U_1U_2U_3\ldots$

where U_i is a gate operation.

(3) Compile - Realization of gates by evolution operations

$$\exp(-iH_i t/\hbar)$$

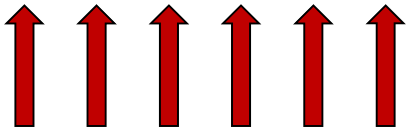
where H_i is a part of a Hamiltonian.

Selective single qubit operation

$$U = \exp(-iHt/\hbar) \quad \text{and} \quad H = \sum_i H_i$$

where $H_i = -\vec{\mu}_i \cdot \vec{H} = -\gamma\hbar H I_{i\alpha} = \hbar\omega I_{i\alpha}$

and I is the spin angular momentum



$$U = \exp(-iH_i t/\hbar)$$

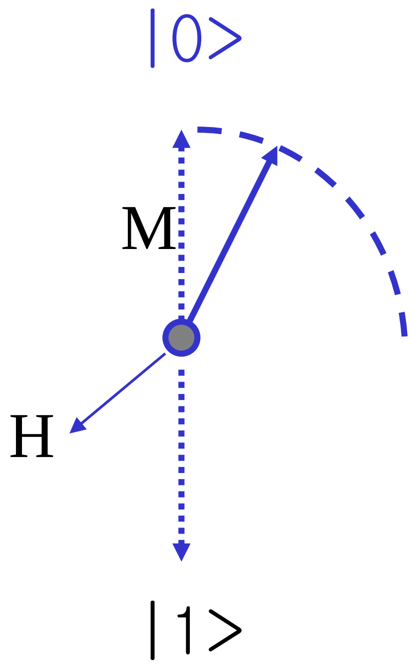
$$= \exp(-i(\hbar\omega_i I_{i\alpha})t/\hbar)$$

$$= \exp(-i\omega_i t I_{i\alpha})$$

$$= \exp(-i\theta I_{i\alpha}) = R_{i\alpha}(\theta)$$

Single qubit operation is performed by pulsed magnetic field

Single qubit operation
in spin quantum computer



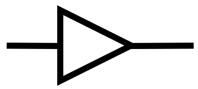
$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

$$= \vec{M} \times \vec{H}_0$$

$$= \gamma \vec{L} \times \vec{H}_0$$

$$\varpi = \gamma H_0$$

Ex) NOT operation



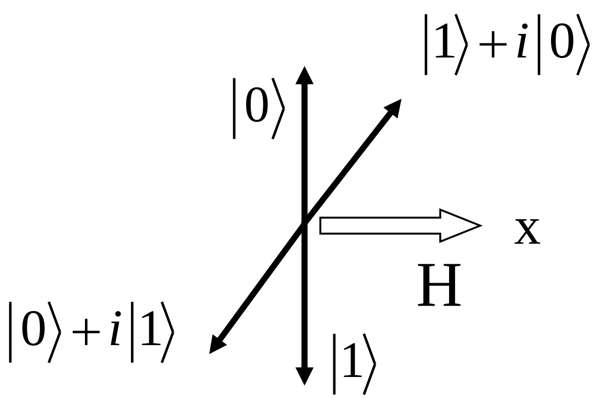
$$|\psi\rangle = \boxed{e^{-iHt/\hbar}}\ |\psi_0\rangle$$

U

Assign $|\uparrow\rangle \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|\downarrow\rangle \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

We need $U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$



$$U = \exp(i\omega t I_x)$$

Set $\omega t = \pi$

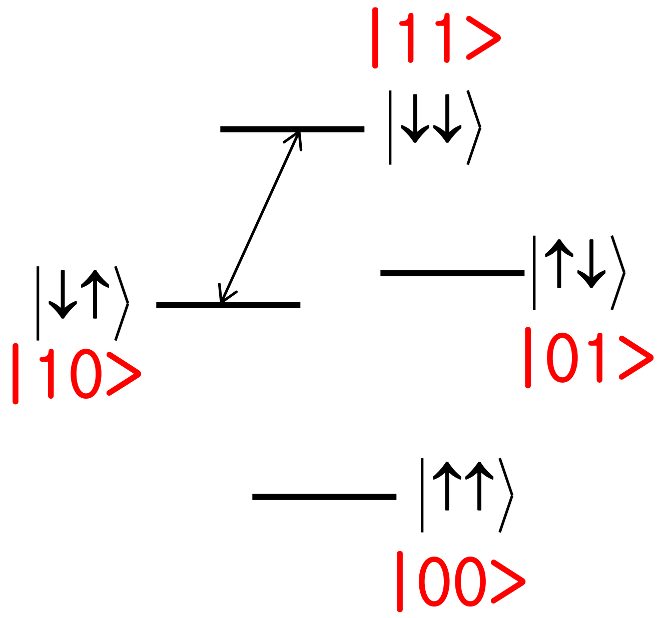
$$U = \exp(i\pi I_x) = \exp(i\frac{\pi}{2}\sigma_x) = i\sigma_x = i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Controlled-NOT

input		output	
C	T	C	T
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

$U(|0\rangle+|1\rangle)|0\rangle$: disentangled state

$=|0\rangle|0\rangle+|1\rangle|1\rangle$: entangled state



C-NOT is performed by Selective excitation

* Controlled-NOT operation

$$U_{C-NOT} =$$

$$R_{1z}(\frac{\pi}{2})R_{2x}(\frac{\pi}{2})R_{2y}(\frac{\pi}{2})U_{12}(-\frac{\pi}{2})R_{2y}(-\frac{\pi}{2})$$

where $R_{i\alpha}(\theta) = \exp(-i\theta I_{i\alpha})$

and

$$U_{ij}(\theta) = \exp(-i(J_{ij}I_{iz}I_{jz})t/\hbar)$$

$$= \exp(-i(J_{ij}t/\hbar)I_{iz}I_{jz})$$

$$= \exp(-i\theta I_{iz}I_{jz})$$

if $H = \sum_{i}\sum_{i,j} J_{ij}I_{iz}I_{jz}$

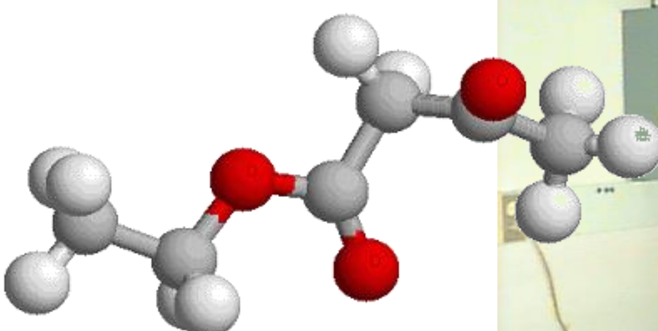
(J-coupling)

Interaction is necessary for C-NOT operation.
C-NOT is performed by just waiting.

3. NMR QC

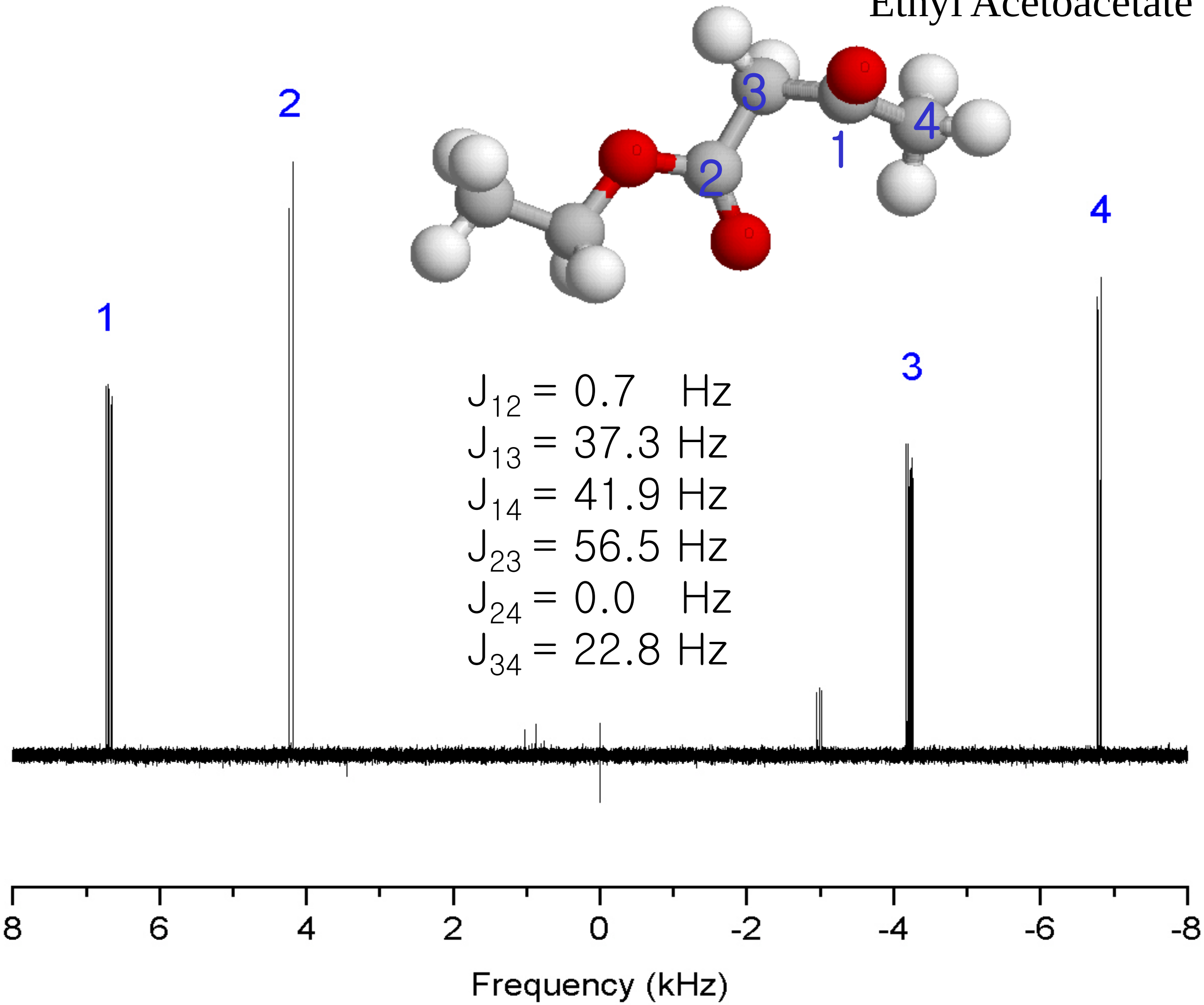
17 years ago…

- 1st demonstration of quantum computing by NMR in 1997



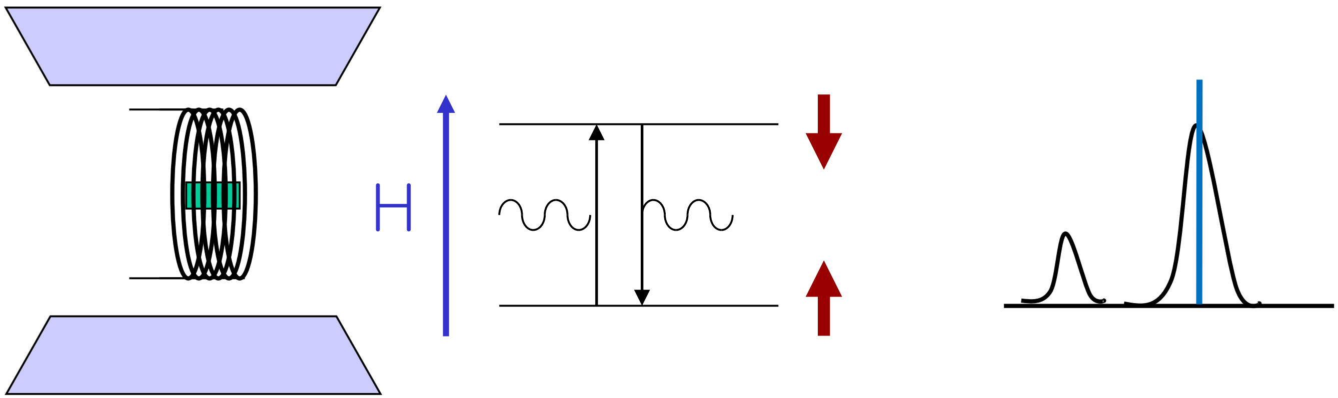
NMR QC

Ethyl Acetoacetate

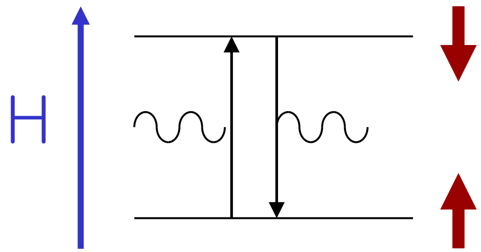


Magnetic Resonance

$$\omega = \gamma H$$

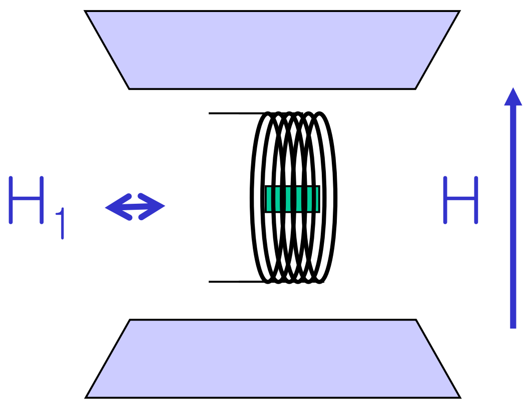


Resonance of magnetic field and electromagnetic wave.
(γ : gyromagnetic ratio)



If transition probability is 1, |↑> ↔ |↓>
What if, ½? |↑> + |↓> ? or |↑> - |↓> ?

In resonance experiment, two magnetic fields are used; one strong static field (H_0), and the other weak rf field (H_1)



Pulse NMR

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

$$= \vec{M} \times \vec{H}_0$$

$$= \gamma \vec{L} \times \vec{H}_0$$

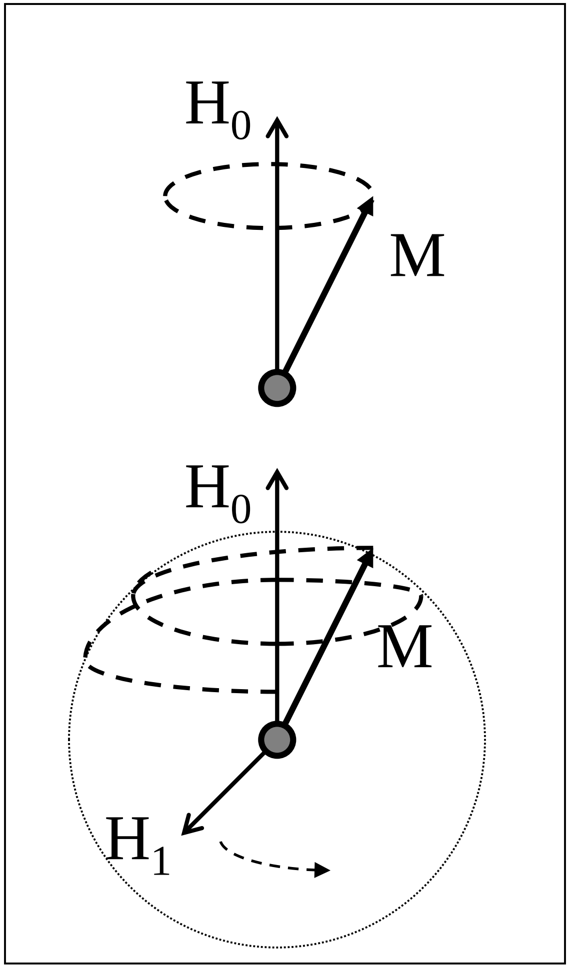
$$\varpi = \gamma H_0$$

(γ : gyromagnetic ratio)

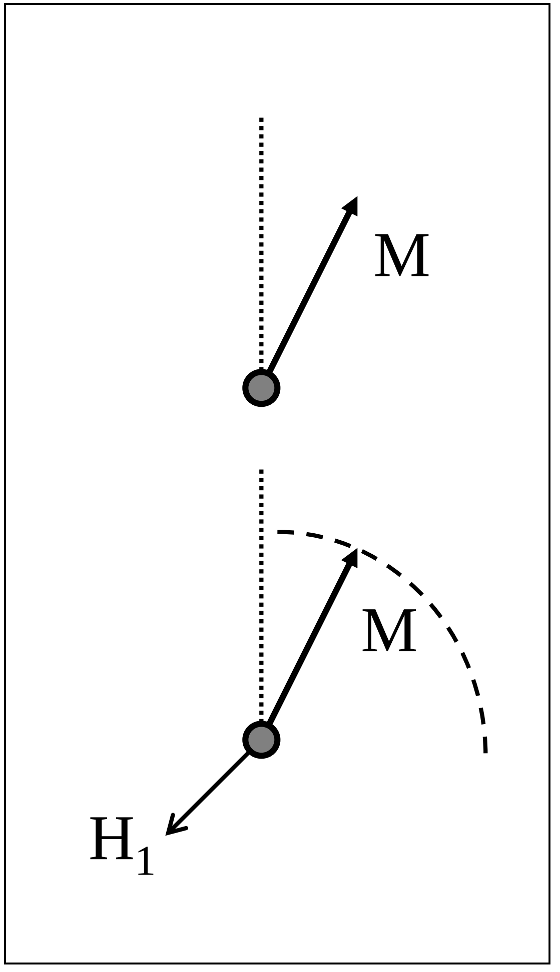
90º pulse

180º pulse

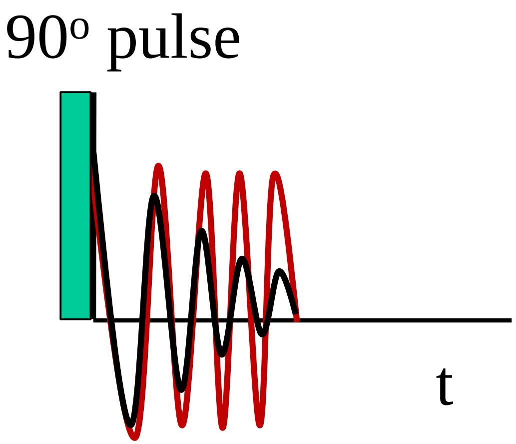
Lab frame

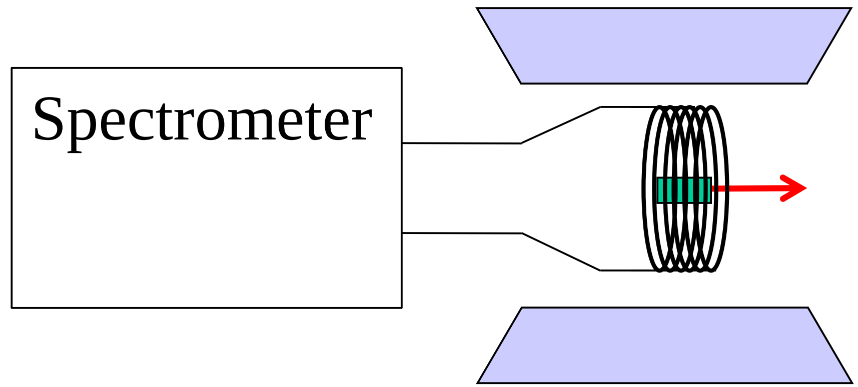


Rotating frame

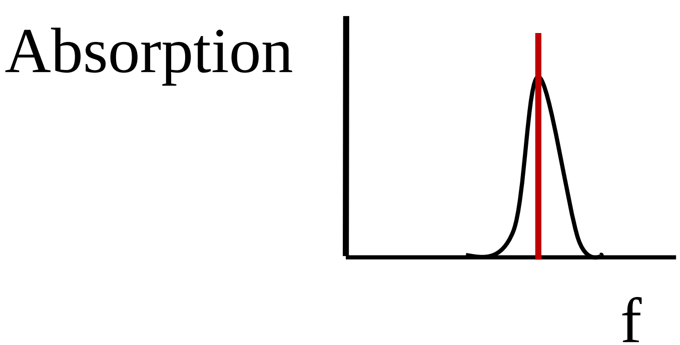


pulse NMR





Spectrometer applies rf pulse and measure free induction decay



Absorption spectrum is the Fourier Transform of the Induction signal.

* Single qubit operation in Quantum Computation

Hamiltonian

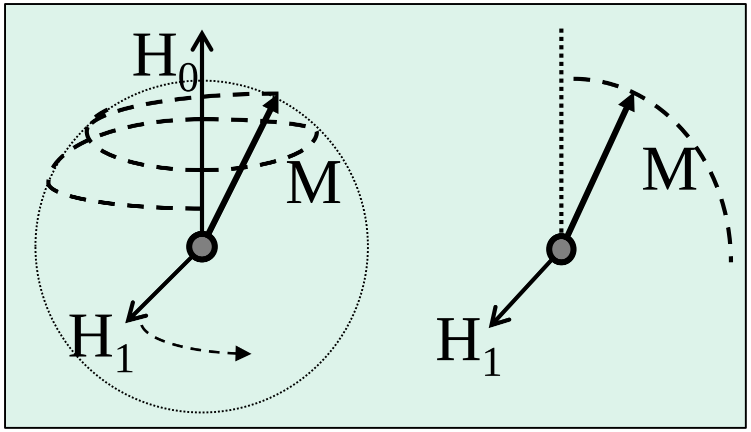
$$H = \mu_\alpha H_0 = \gamma L_\alpha H_0 = \hbar \omega I_\alpha$$

Evolution

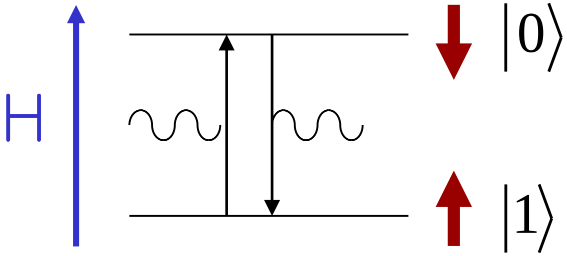
$$R_\alpha(\theta) = \exp(-iHt/\hbar)$$

$$= \exp(-i\omega t I_\alpha) = \exp(-i\theta I_\alpha)$$

Rotation operator



Single qubit operation in NMR is performed by an rf pulse.



* Controlled-NOT operation

$$U_{C-NOT} =$$

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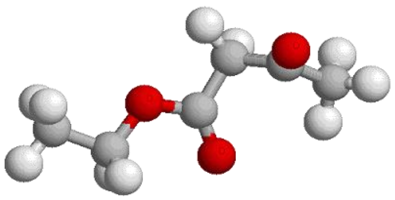
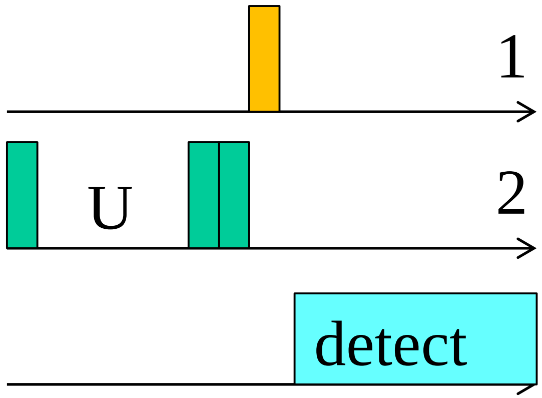
where $R_{i\alpha}(\theta) = \exp(-i\theta I_{i\alpha})$

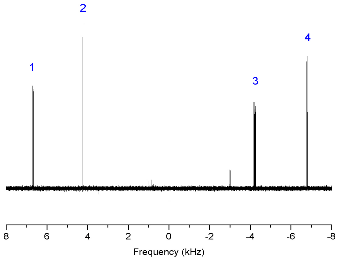
and
$$U_{ij}(\theta) = \exp(-i(J_{ij}I_{iz}I_{jz})t/\hbar)$$
$$= \exp(-i(J_{ij}t/\hbar)I_{iz}I_{jz})$$
$$= \exp(-i\theta I_{iz}I_{jz})$$

if $H = \sum_i \sum_{i,j} J_{ij}I_{iz}I_{jz}$

(J-coupling)





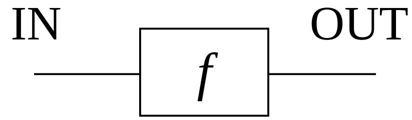


Interaction is necessary for C-NOT operation

4. Example : Deutsch Algorithm

Refined **Deutsch**'s Algorithm

1 bit function *f(x)*



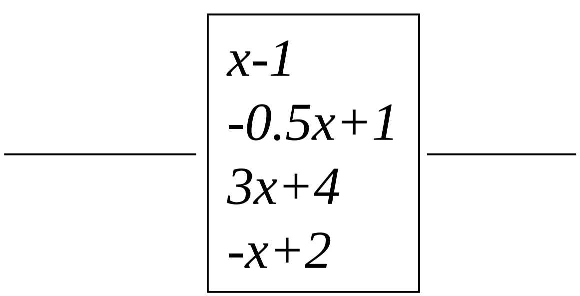
IN	OUT			
	f_{00}	f_{01}	f_{10}	f_{11}
0	0	0	1	1
1	0	1	0	1

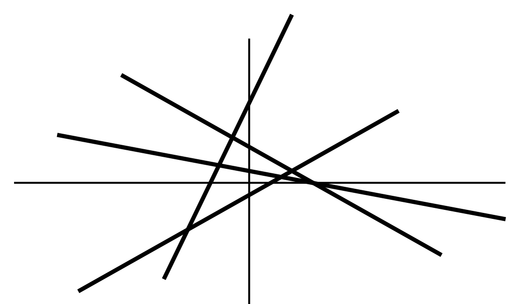
f_{00}, f_{11}:constant fn

f_{01}, f_{10}:balanced fn

- Problem: Is a given function *f* balanced or constant?
- To answer, classical computing requires 2 operations, f(0) & f(1).

cf)





For 2 qubits, 3 operations are required classically

IN	OUT															
00	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
01	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
10	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
11	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

For n qubits, there are 2^n input states & $2^{\wedge 2n}$ output states.
$2^{n-1}+1$ operations are required classically.

•Number state in QC : |0>, |1>

ex) 0 + 1 = 1, but $|0\rangle + |1\rangle \neq |1\rangle$

	f_{00}	f_{01}	f_{10}	f_{11}
0	0	0	1	1
1	0	1	0	1

⇨

	f_{00}	f_{01}	f_{10}	f_{11}
|0>	|0>	|0>	|1>	|1>
|1>	|0>	|1>	|0>	|1>

• QC requires only 1 operation (irrespective of n) iff

(i) initial state is |0>+|1>

(ii) (unitary) operation U : $|x\rangle \xrightarrow{U} (-1)^{f(x)}|x\rangle$

then, $|0\rangle + |1\rangle \xrightarrow{U_{00}} (-1)^{f_{00}(0)}|0\rangle + (-1)^{f_{00}(1)}|1\rangle$

$$= (-1)^0|0\rangle + (-1)^0|1\rangle$$

$$= |0\rangle + |1\rangle$$

	f_{00}	f_{01}	f_{10}	f_{11}
|0>	|0>	|0>	|1>	|1>
|1>	|0>	|1>	|0>	|1>

$$|0\rangle + |1\rangle \xrightarrow{U_{00}} (-1)^{f_{00}(0)}|0\rangle + (-1)^{f_{00}(1)}|1\rangle = |0\rangle + |1\rangle$$

$$|0\rangle + |1\rangle \xrightarrow{U_{01}} (-1)^{f_{01}(0)}|0\rangle + (-1)^{f_{01}(1)}|1\rangle = |0\rangle - |1\rangle$$

$$|0\rangle + |1\rangle \xrightarrow{U_{10}} (-1)^{f_{10}(0)}|0\rangle + (-1)^{f_{10}(1)}|1\rangle = -|0\rangle + |1\rangle$$

$$|0\rangle + |1\rangle \xrightarrow{U_{11}} (-1)^{f_{11}(0)}|0\rangle + (-1)^{f_{11}(1)}|1\rangle = -|0\rangle - |1\rangle$$

Balanced functions change relative phase.

Parallel processing thanks to superposition principle!

Implementation of 1 qubit Deutsch's algorithm

(1) Preparation – make |0> (or |1>) state.

(2) Superposition

– pseudo-Hadamard operation $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

(3) (Unitary) Operations

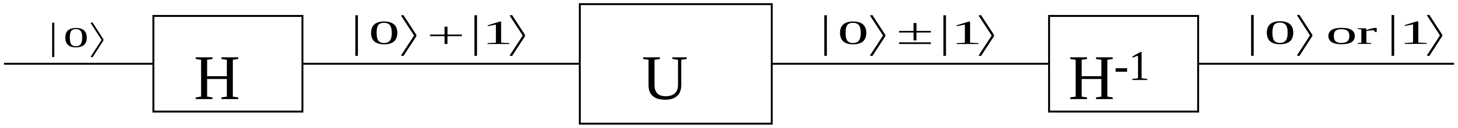
$$U_{00} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, U_{01} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, U_{10} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, U_{11} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

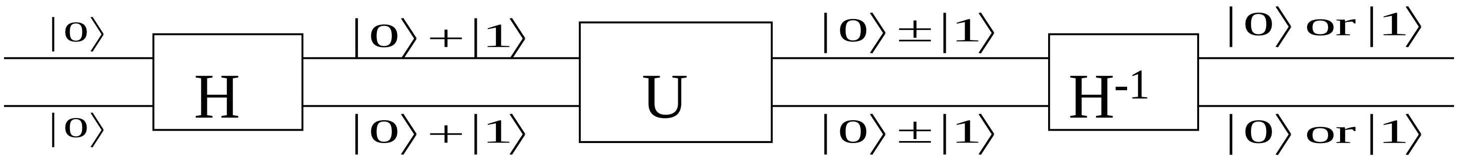
(4) Inverse pseudo-Hadamard $H^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(5) Reading

Quantum network





$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Operation

$$U_{00} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad : \text{Do nothing}$$

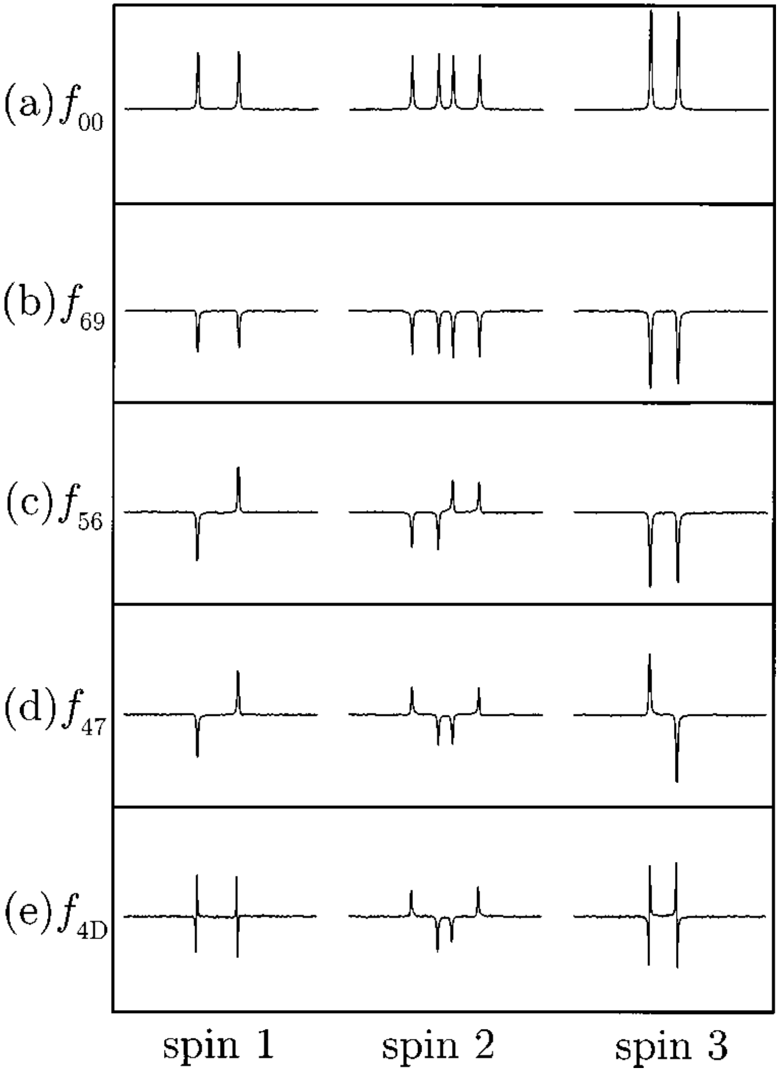
$$U_{11} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -U_{00}$$

$$U_{01} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\equiv R_x(180°)R_y(180°) = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = -i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$U_{10} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = -U_{01}$$

Implementation of the refined Deutsch-Jozsa algorithm on a three-bit NMR quantum computer



2^^{2n} output states
N=3
2^8=64 different functions

5. Bottleneck of QC

Quantum systems suggested as QC

Atomic and Molecular

Ion trap
Cavity QED
NMR
Molecular magnet
N@C_{60}(fullerine)
BEC

Optical

Photon
Photonic crystal

Solid State

Quantum dot
Superconductor
Si-based QC

Electron beam

el. floating on liquid He
el. trapped by SAW
el. trapped by magnetic field

Model quantum computer

- Hamiltonian – Zeeman & interaction terms.

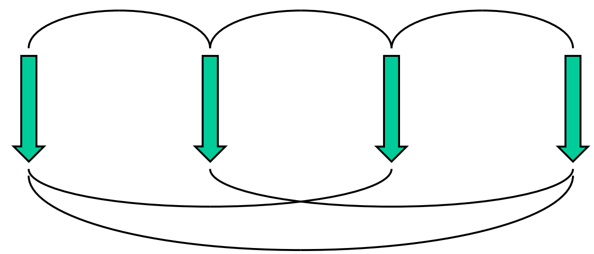
$$H = \sum_i \hbar\omega_i I_{i\alpha} + \sum_{i,j} J_{ij} I_{iz} I_{jz}$$

-single qubit is realized by Zeeman term : Apply magnetic field and wait
-controlled-NOT is realized by interaction term :Wait
-What if the Hamiltonian is different?
-Interactions other than Ising type are valid?

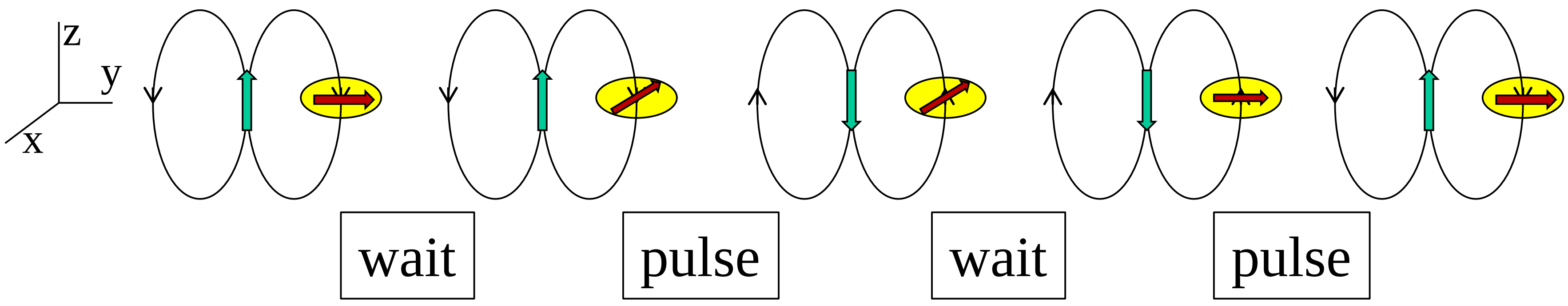
- Turn on and off each term independently
  - addressing & interaction control
  - Can we turn off the interaction?

- Interaction control
  - Turn on only selected interactions, or turn off unwanted interactions

$$\sum_{i,j}^{4} J_{ij} I_i I_j = J_{12} I_1 I_2 + J_{13} I_1 I_3 + J_{14} I_1 I_4 + J_{23} I_2 I_3 + J_{24} I_2 I_4 + J_{34} I_3 I_4$$

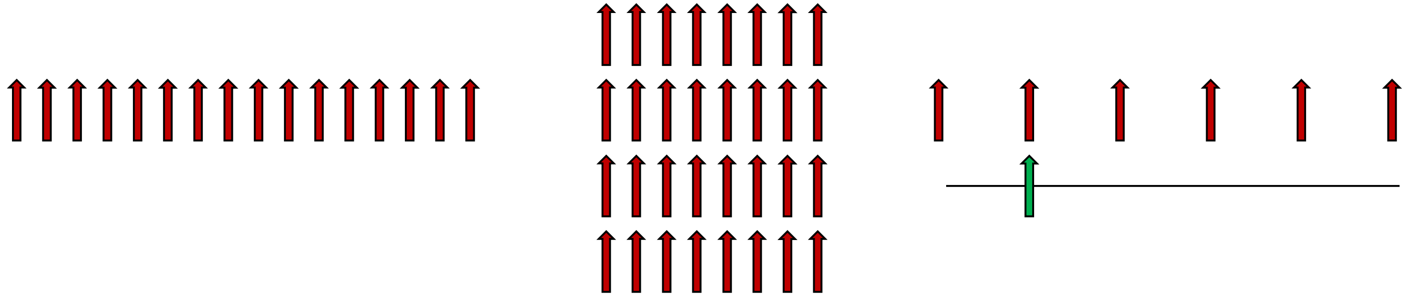


  - Refocusing sequence – effectively turn off interactions
    - Hamiltonian Engineering



Interaction control is the real challenge

- Refocusing pulse sequence increase exponentially with # of qubits
- How can we make far qubits interact?
  - Moving qubit



Conclusion

- Making a quantum computer is the bottleneck in the development of QIT.
- New quantum computer systems are being suggested.
- In building practical quantum computers, interaction control is the bottleneck.

END